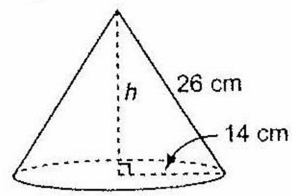
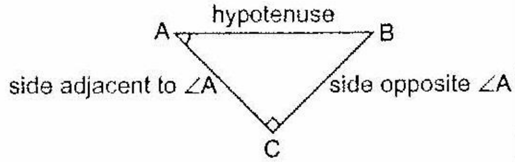
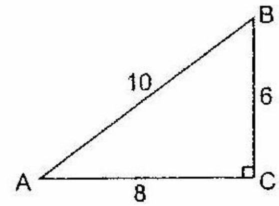
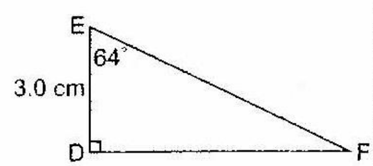
Math 10
Final Exam Course Review

Chapter 1 TOPICS

Skill	Description	Example
Convert between units in the imperial system.	Use the relationships between the units.	Convert: 19 yd. to feet 1 yd. = 3 ft. 19 yd. = 19 × 3 ft. 19 yd. = 57 ft.
Convert between units in the imperial system and the SI system.	Use the relationships between the systems.	Convert: 37 mi. to kilometres 1 mi. ≐ 1.6 km 37 mi. ≐ 37 × 1.6 km 37 mi. ≐ 59.2 km
Find the slant height or height of a cone.	Use the Pythagorean Theorem.	To the nearest centimetre, find the height of this cone.  $26^2 = 14^2 + h^2$ $h^2 = 26^2 - 14^2$ $h = \sqrt{26^2 - 14^2}$ $h = 21.9089...$ The height is about 22 cm.
Find the surface areas of pyramids, prisms, cones, cylinders, and spheres.	For a pyramid or a prism, add the area of the base or bases to the area of the faces. For a cone or a cylinder, add the area of the base or bases to the curved surface area. Surface area of a sphere is: $4\pi(\text{radius})^2$	To the nearest square foot, find the surface area of a cylinder with radius 3 ft. and height 9 ft. $SA = 2\pi r^2 + 2\pi rh$ $SA = 2\pi(3)^2 + 2\pi(3)(9)$ $SA = 226.1946...$ The surface area is about 226 square feet.
Find the volumes of prisms, cylinders, pyramids, cones, and spheres.	Volume of a prism and a cylinder is: (base area)(height) Volume of a pyramid and a cone is: $\frac{1}{3}$(base area)(height) Volume of a sphere is: $\frac{4}{3}\pi(\text{radius})^3$	To the nearest cubic inch, find the volume of a cylinder with radius 2 in. and height 8 in. $V = \pi r^2 h$ $V = \pi(2)^2(8)$ $V = 100.5309...$ The volume is about 101 cubic inches.

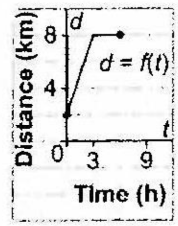
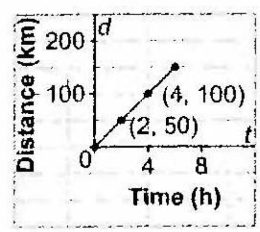
Chapter 2 TOPICS

Skill	Description	Example
Find a trigonometric ratio.	In $\triangle ABC$,  $\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$ $\cos A = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\tan A = \frac{\text{opposite}}{\text{adjacent}}$	 $\sin A = \frac{\text{opposite}}{\text{hypotenuse}}$ $\sin A = \frac{BC}{AB}$ $\sin A = \frac{6}{10}$, or 0.6
Find the measure of an angle.	To find the measure of an acute angle in a right triangle: **1.** Use the given lengths to write a trigonometric ratio. **2.** Use the inverse function on a scientific calculator to find the measure of the angle.	To find the measure of $\angle B$ in $\triangle ABC$ above: $\tan B = \frac{\text{opposite}}{\text{adjacent}}$ $\tan B = \frac{AC}{BC}$ $\tan B = \frac{8}{6}$ $\angle B = \tan^{-1}\left(\frac{8}{6}\right)$ $\angle B \doteq 53°$
Find the length of a side.	To find the length of a side in a right triangle: **1.** Use the measure of an angle and the length of a related side to write an equation using a trigonometric ratio. **2.** Solve the equation.	To find the length of EF in $\triangle DEF$:  $\cos E = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\cos E = \frac{DE}{EF}$ $\cos 64° = \frac{3.0}{EF}$ $EF \cos 64° = 3.0$ $EF = \frac{3.0}{\cos 64°}$ $EF = 6.8435\ldots$ $EF \doteq 6.8$ cm

Skill	Description	Example
Write the prime factorization of a number.	Write the prime factors of a number as a product.	The prime factorization of 18 is: $2 \times 3 \times 3$, or 2×3^2
Find the greatest common factor (GCF) of two numbers.	List the factors of each number. Identify the greatest factor that is in both lists.	Find the GCF of 12 and 16: Factors of 12: 1, 2, 3, **4**, 6, 12 Factors of 16: 1, 2, **4**, 8, 16 The GCF of 12 and 16 is 4.
Find the least common multiple (LCM) of two numbers.	List multiples of each number. Identify the least multiple that is in both lists.	Find the LCM of 12 and 16: Multiples of 12: 12, 24, 36, **48**, 60, … Multiples of 16: 16, 32, **48**, 64, 80, … The LCM of 12 and 16 is 48.
Find the square root of a perfect square and the cube root of a perfect cube.	Group the prime factors.	Find $\sqrt{64}$: $64 = (2 \times 2 \times 2) \times (2 \times 2 \times 2)$ $\sqrt{64} = 2 \times 2 \times 2$, or 8 Find $\sqrt[3]{64}$: $64 = (2 \times 2) \times (2 \times 2) \times (2 \times 2)$ $\sqrt[3]{64} = 2 \times 2$, or 4
Factor polynomials with a common factor.	Find the GCF of the terms. Divide to find the other factor.	Factor $6n^2 - 18n$: The GCF of $6n^2$ and $18n$ is $6n$. $6n^2 - 18n = 6n(n - 3)$
Multiply polynomials.	Multiply each term in one polynomial by each term in the other polynomial.	Multiply $(b - 4)(2b + 3)$: $(b - 4)(2b + 3)$ $= b(2b + 3) - 4(2b + 3)$ $= 2b^2 + 3b - 8b - 12$ $= 2b^2 - 5b - 12$
Factor trinomials.	Factor the 1st and 3rd terms. Write the possible binomials.	Factor $x^2 + 4x + 3$: List factors of 3: (1, 3) $x^2 + 4x + 3 = (x + 1)(x + 3)$
Factor special polynomials.	Factor a difference of squares and a perfect square trinomial.	Factor $4a^2 - 9$: $4a^2 - 9 = (2a)^2 - 3^2$ $4a^2 - 9 = (2a + 3)(2a - 3)$ Factor $9m^2 + 24m + 16$: Write the 1st and 3rd terms as perfect squares. $9m^2 = (3m)^2$ and $16 = 4^2$ $9m^2 + 24m + 16 = (3m + 4)(3m + 4)$ $= (3m + 4)^2$

Chapter 4 TOPICS

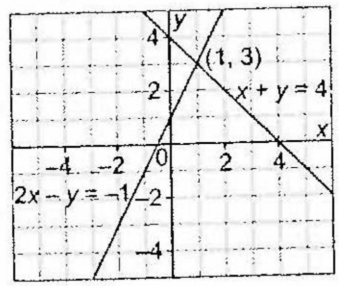
Skill	Description	Example
Classify radicals as rational or irrational numbers.	A radical is rational if it can be written as: a fraction, a terminating decimal, or a repeating decimal.	Classify each radical as rational or irrational. $\sqrt{2.25} = 1.5$, so $\sqrt{2.25}$ is rational $\sqrt{2} = 1.4142\ldots$, so $\sqrt{2}$ is irrational
Simplify a radical.	Write the radicand as a product with a perfect power as one factor. Then write the radical as the product of a whole number and a radical.	Simplify. $\sqrt{24}$ $\sqrt{24} = \sqrt{4 \cdot 6}$ $= \sqrt{4} \cdot \sqrt{6}$ $= 2\sqrt{6}$
Write a mixed radical as an entire radical.	Write the whole number as the root of a perfect power. Then multiply the perfect power and the radicand to get the new radicand.	Express $2\sqrt[3]{5}$ as an entire radical. $2 = \sqrt[3]{2 \cdot 2 \cdot 2}$, or $\sqrt[3]{8}$ So, $2\sqrt[3]{5} = \sqrt[3]{8} \cdot \sqrt[3]{5}$ $= \sqrt[3]{8 \cdot 5}$ $= \sqrt[3]{40}$
Write a power with a rational number exponent as a radical.	The numerator of the exponent is the power to which the base is raised. The denominator of the exponent is the index of the radical.	Express $7^{\frac{2}{3}}$ as a radical. $7^{\frac{2}{3}} = \sqrt[3]{7^2}$ or $7^{\frac{2}{3}} = \left(\sqrt[3]{7}\right)^2$
Write a power with a negative exponent as a power with a positive exponent.	Write the reciprocal of the base, and change the exponent from negative to positive.	Write each power with a positive exponent. $\left(\frac{3}{5}\right)^{-4} = \left(\frac{5}{3}\right)^4$ $2^{-3} = \frac{1}{2^3}$
Apply exponent laws to simplify expressions.	The exponent laws apply for rational and integer exponents and bases, and for variable bases. The expression is usually written with positive exponents.	Simplify. $x^{-\frac{1}{2}} \cdot x^{-2} = x^{-\frac{1}{2}-2}$ $= x^{-\frac{1}{2}-\frac{4}{2}}$ $= x^{-\frac{5}{2}}$, or $\frac{1}{x^{\frac{5}{2}}}$

Skill	Description	Example
Find the domain and range of a function.	The domain is the set of 1st elements of the ordered pairs in a function. The range is the set of related 2nd elements. For a graph, the domain is represented by the shadow of the graph on the horizontal axis. The range is represented by the shadow of the graph on the vertical axis.	For the set of ordered pairs: $\{(-1, 4), (0, 6), (1, 8)\}$ The domain is: $\{-1, 0, 1\}$ The range is: $\{4, 6, 8\}$ For the graph:  The domain is: $0 \leq t \leq 6$ The range is: $2 \leq d \leq 8$
Find the rate of change from the graph of a linear function.	The rate of change of a linear function is: $\frac{\text{change in dependent variable}}{\text{change in independent variable}}$ The rate of change is positive when the graph goes up to the right, and negative when the graph goes down to the right.	**Distance against Time**  Rate of change $= \frac{100\text{ km} - 50\text{ km}}{4\text{ h} - 2\text{ h}}$ $= \frac{50\text{ km}}{2\text{ h}}$ $= 25$ km/h
Find the intercepts of the graph of a linear function.	To find the x-intercept, substitute $y = 0$. To find the y-intercept, substitute $x = 0$.	For the linear function $y = -2x + 8$: When $y = 0$, $0 = -2x + 8$ $2x = 8$ $x = 4$ The x-intercept is 4. When $x = 0$, $y = -2(0) + 8$ $y = 8$ The y-intercept is 8.

Chapter 6 TOPICS

Skill	Description	Example
Find the slope of a line.	Slope $= \dfrac{\text{rise}}{\text{run}}$ The slope of a line through $A(x_1, y_1)$ and $B(x_2, y_2)$ is: $\dfrac{y_2 - y_1}{x_2 - x_1}$	For $A(2, -4)$ and $B(-1, 3)$: Rise: $3 - (-4) = 7$ Run: $-1 - 2 = -3$ Slope: $\dfrac{7}{-3}$, or $-\dfrac{7}{3}$
Identify parallel lines and perpendicular lines.	Parallel lines have equal slopes. Perpendicular lines have slopes that are negative reciprocals.	Line AB has slope $-\dfrac{7}{3}$. Line CD has slope $-\dfrac{7}{3}$. Line EF has slope $\dfrac{3}{7}$. Lines AB and CD are parallel. Lines AB and EF are perpendicular. Lines CD and EF are perpendicular.
Write the equation of a line in slope-intercept form.	A line with slope, m, and y-intercept, b, has equation: $y = mx + b$	For a line with slope 3 and y-intercept -2, an equation is: $y = 3x - 2$
Write the equation of a line in slope-point form.	A line with slope, m, that passes through $P(x_1, y_1)$, has equation: $y - y_1 = m(x - x_1)$	A line with slope -4 that passes through $P(-1, 3)$ has equation: $y - 3 = -4(x - (-1))$, or $y - 3 = -4(x + 1)$
Find the intercepts of a line when its equation is in general form.	The general form of an equation is: $Ax + By + C = 0$, where A, B, and C are integers, and A is positive	A line has equation: $3x - 2y + 6 = 0$ For the y-intercept, substitute $x = 0$: $3(0) - 2y + 6 = 0$ $-2y = -6$ $y = 3$ For the x-intercept, substitute $y = 0$: $3x - 2(0) + 6 = 0$ $3x + 6 = 0$ $3x = -6$ $x = -2$

Chapter 7 TOPICS

Skill	Description	Example
Solve a linear system by graphing.	To solve a linear system, graph both equations on the same grid or enter the equations in a graphing calculator. The coordinates of the point of intersection are the solution of the linear system.	For this linear system: $2x - y = -1$ $x + y = 4$  The solution of the system is: $x = 1$ and $y = 3$
Solve a linear system using algebra.	To solve a linear system, use substitution or elimination. Then verify that the solution is correct by substituting the x- and y-values into both equations.	For this linear system: $2x + y = 3$ ① $3x - y = 7$ ② The coefficients of y are opposite integers, so use elimination. $2x + y = 3$ ① $+(3x - y = 7)$ ② $5x = 10$ $x = 2$ Substitute $x = 2$ in equation ①. $2(2) + y = 3$ $y = -1$ The solution is: $x = 2$ and $y = -1$
Find the number of solutions of a linear system.	To find the number of solutions of a linear system: • compare the graphs of the equations, or • compare the slopes and y-intercepts of the lines	These lines have different slopes, so the system has exactly one solution: $y = 3x + 4$ $y = -2x + 1$ These lines have the same slope and different y-intercepts, so the system has no solution: $y = 3x + 4$ $y = 3x + 2$ These lines have the same slope and the same y-intercept, so the system has infinite solutions: $y = 3x + 4$ $2y = 6x + 8$

Math 10
Final Exam Course Review – January 2011

A: Conversions and Measurement FP10.3

1. <u>Convert each of the following</u>

(a) convert 1458 m to miles (b) convert 3 yd 2 ft to cm.

<u>Solving Problems Involving Objects</u>

1. A hemisphere has a diameter of 10 feet.

(a) What is the surface area of this hemisphere, to the nearest square foot?

(b) What is the volume of the hemisphere, the nearest cubic foot?

2. A right rectangular pyramid has a base with dimensions 8m by 7m, and height of 6 m. Determine the surface area of this pyramid to the nearest square m.

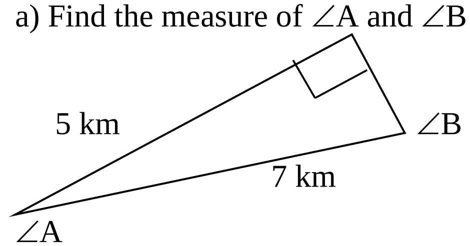
3. A bowl of sugar was knocked over. The spilled sugar formed a cone with a radius of 4 cm and a slant height of 6 cm. How much sugar was in the pile?

B: Trigonometry

<u>Determine side and angle measurements</u>

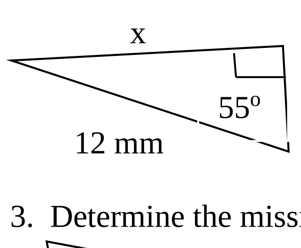
1. Refer to the right triangle below. FP10.4

a) Find the measure of $\angle A$ and $\angle B$ to the nearest degree.

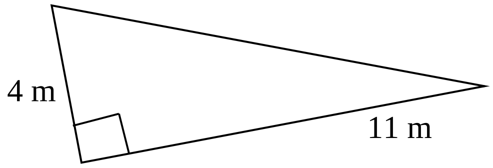


b) Now find the length of the missing side.

2. Determine the length of x:



3. Determine the missing angles:



4. Find the values of the following to 4 decimal places using your calculator:

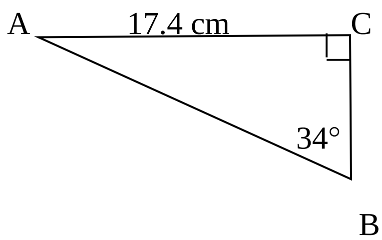
(a) sin 34° = _________________

(b) cos 58° = _________________

(c) tan 46° = _________________

5. If cos θ = 0.5402, find the measure of angle θ rounded to one decimal place.

6. Solve the triangle found below.



m ∠ A = __________

m ∠C = __________

$\overline{AB}$ = __________

$\overline{BC}$= __________

Problem Solving involving one or more right Triangles

1. A flagpole casts a shadow that is 25 m long when the angle of between the sun’s ray and the ground is 40 degrees. What is the height of the flagpole to the nearest meter?
2. An escalator is 14.5 m long. The escalator makes an angle of 27 degrees with the ground. What is the height of the escalator? Give your answer to the nearest tenth of a meter.

C: Factors and Products

1. Expand and simplify

(a) $(2x+8)(3x-2)$ (b) $5(x-3)^2$ (c) $(4 + 3)^2$

2. Factor (use prime factorization)

(a) 324 (b) 120

3. Find the Greatest Common Factor

(a) 12 and 18 (b) 24 and 60

4. Find the Least Common Multiple

(a) 12 and 15 (b) 16 and 20

5. Perfect Squares and Cube Roots

(a) Using Prime Factorization find the square and cube root of 64

6. Factor polynomials using a common factor

(a) 6n2 – 18n (b) $6x^3y^2 + 2xy^5$

7. Factor Trinomials

(a) $(a^2+7a-18)$ (b) x^2-8x+7 (c) $3v^2-8v+4$

8. Factor Difference of Squares and Perfect Square Trinomials

(a) d^2 - 16 (b) $w^2 - 14w + 49$

9. Factor out a common factor then use difference of squares

(a) $8x^2$-$72y^2$

D: Roots and Powers

1. Use prime factorization to simplify

(a) $\sqrt{72}$ (b) $\sqrt[4]{256}$ (b) $\sqrt{147}$

2. Write as an Entire radical

(a) $2\sqrt{7}$ (b) $5\sqrt[3]{3}$

3. Estimate to 1 decimal place

(a) $\sqrt{52}$ (b) $\sqrt[3]{30}$

4. Are the following Rational or Irrational Numbers

(a) $\sqrt{\frac{4}{9}}$ (b) $\sqrt[2]{17}$

5. Evaluate

(a) $36^{\frac{-1}{2}}$ (b) $27^{\frac{2}{3}}$ (c) $\left(\sqrt{\frac{4}{9}}\right)^{-3}$

6. Write Power as a radical

(a) $15^{\frac{2}{3}}$

7. Write as a power with a fractional exponent

(a) $\sqrt[3]{5^2}$

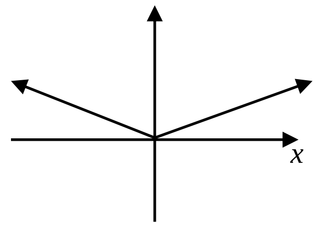
8. Multiply and Divide

(a) $2^5 \bullet 2^3$ (b) $x^{\frac{-1}{2}} \bullet x^{-2}$

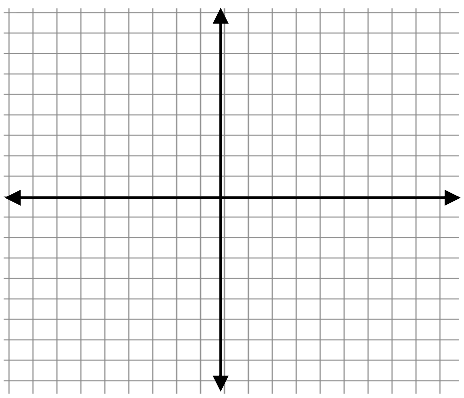
(c) $\frac{(4xy)^2}{2y}$

E: Functions and Relations

1. Refer to the graph to the right. Is this relation a function? Why or why not?



2. <u>Graph $6x + 2y = 12$ using *x* and *y* intercepts</u>



<u>*x-int*</u> <u>*y-int*</u>

3. <u>Relations.</u> Answer the questions for the following 2 tables

Athlete	**Sport**
Crosby	Hockey
Jones	Curling
Wotherspoon	Speed Skating
Ovechkin	Hockey

a) Why is this relation a function?

b) Write the domain and range. Explain any restrictions on them.

c) Represent this relation as an arrow diagram.

Number of Minutes n	**Cost, C ($)**
10	2
20	4
30	6
40	8
50	10

d) Why is this relation a function?

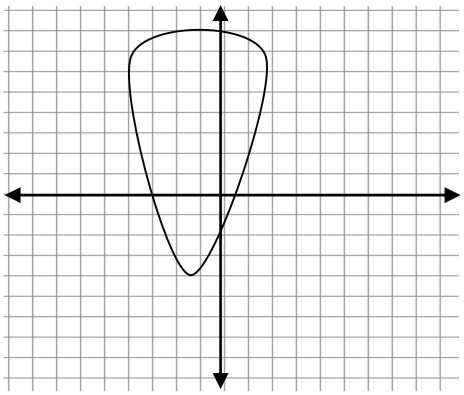
e) Write the domain and range. Explain any restrictions on them.

f) Identify the dependent and independent variables.

g) Does this table represent a linear relation? How could you find out mathematically? From a graph?

4. Find the domain and range for each graph. Are the following Functions?

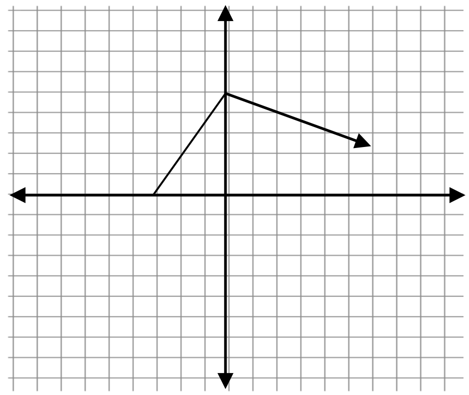
(a)



D =

R =

(b)



D =

R =

5. Function Notation

Carmen works for a research company. The equation P= 5n + 30 represents her daily pay, P dollars, when she conducts n surveys.

a) Describe the function. Write the equations using function notation.

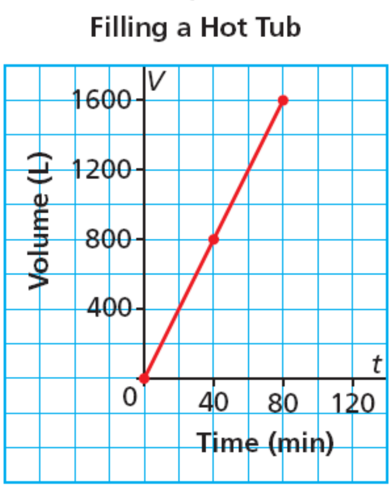
b) Find the the value of P(8). What does this number represent?

c) Find the value of n when P(n) = 90. What does this number represent?

6. Table of values

Create a table of values for **y = x + 2** and answer whether it represents a linear relation.

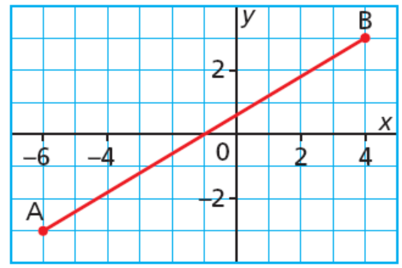
7. Rate of Change



a) Identify the dependent and independent variables.
b) Determine the rate of change of each relation, then describe what it represents.

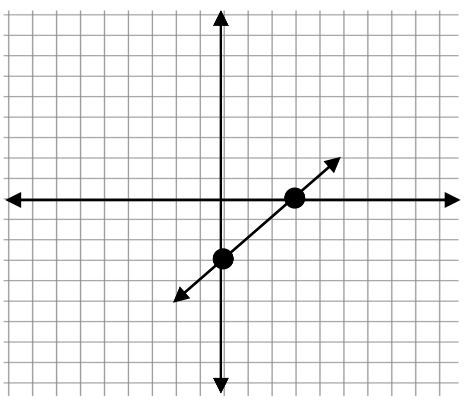
F: Linear Functions **FP10.6-9**

1. Graph the following linear function in the space provided using the slope and *y*-intercept method only. State the slope and *y*-intercept first. $y = \frac{3}{5}x - 4$



2. Slope
   (a) Find the slope of a line that passes through (2, -3) and (-4, 3)
   (b) Find the slope from the graph to the right.
   (c) Graph a line whose slope is $\frac{-2}{3}$ and goes through (3, 2)
   (d) Are y = 2x - 3 and $y = \frac{-1}{2}x - 4$ parallel or perpendicular lines? Explain the difference.

3. Write the equation of the line in the specified form
   (a) Write the equation of the line in point-slope form whose slope is 3 and passes through the point (2, 5)
   (b) Write the equation of the line in general form that passes through (0, 3) and (2, 3)
   (c) Write an equation in slope-intercept form for the line in the graph below:



   (d) Now write that same equations from questions (a), (b), and (c) in standard form and general form.
   (e) Write the equation of a line in Slope-Point form that has a slope of $\frac{-3}{4}$ and passes through (2, -3)
   (f) Write the equation of a line in Slope-Point form that passes through (-3,5) and (3,1)
   (g) Write the equation $y = \frac{1}{2}x - 2$ in Standard form and then into general form.